Two gaps with one energy scale in cuprate superconductors

Shiping Feng*, Huaisong Zhao, and Zheyu Huang
Department of Physics, Beijing Normal University, Beijing 100875, China

The interplay between the superconducting gap and normal-state pseudogap in cuprate superconductors is studied based on the kinetic energy driven superconducting mechanism. It is shown that the interaction between charge carriers and spins directly from the kinetic energy by exchanging spin excitations induces the normal-state pseudogap state in the particle-hole channel and superconducting-state in the particle-particle channel, therefore there is a coexistence of the superconducting gap and normal-state pseudogap in the whole superconducting dome. This normal-state pseudogap is closely related to the quasiparticle coherent weight, and is a necessary ingredient for superconductivity in cuprate superconductors. In particular, both the normal-state pseudogap and superconducting gap are dominated by one energy scale, and they are the result of the strong electron correlation.

PACS numbers: 74.20.Mn, 74.25.Dw, 74.72.Kf

After intensive investigations over more than two decades, it has become clear that the interplay between the superconducting (SC) gap and normal-state pseudogap is one of the most important problems in cuprate superconductors[2,3]. The parent compounds of cuprate superconductors are believed to belong to a class of materials known as Mott insulators with an antiferromagnetic (AF) long-range order (AFLRO), where a single common feature is the presence of the CuO_2 plane[4]. As the CuO_2 planes are doped with charge carriers, the AF phase subsides and superconductivity emerges[4], then the physical properties mainly depend on the extent of doping, and the regimes have been classified into the underdoped, optimally doped, and overdoped, respectively. Experimentally, a large body of experimental data available from a wide variety of measurement techniques have provided rather detailed information on the normal-state pseudogap state and SC-state in cuprate superconductors[2–13], where an agreement has emerged that the normal-state pseudogap state is particularly obvious in the underdoped regime, i.e., the magnitude of the normal-state pseudogap is much larger than that of the SC gap in the underdoped regime, then it smoothly decreases upon increasing the doping concentration. This is why the normal-state properties in the underdoped regime exhibit a number of the anomalous properties[14]. However, there is a controversy about the phase diagram with respect to the normal-state pseudogap line[2]. On the one hand, some authors[15] analyzed the experimental data, and then they argued that the normal-state pseudogap line intersects the SC dome at about optimal doping. On the other hand, it has been argued that the normal-state pseudogap merges gradually with the SC gap in the overdoped regime[16], eventually disappearing together with superconductivity at the end of the SC dome. Theoretically, to the best of our knowledge, all theoretical studies of the normal-state pseudogap phenomenon and its relevance to superconductivity performed so far are based on the phenomenological d-wave Bardeen-Cooper-Schrieffer (BCS) formalism[15–19], where by introducing a phenomenological doping and temperature dependence of the normal-state pseudogap, the two-gap feature in cuprate superconductors is reproduced[19]. In particular, a phenomenological theory of the normal-state pseudogap state has been developed[20], where an ansatz is proposed for the coherent part of the single particle Green's function in a doped resonant valence bond state, and then the calculated result of the electronic properties in the normal-state pseudogap phase is in qualitative agreement with the experimental data. Moreover, it has been argued recently that the pseudogap is a combination of a quantum disordered d-wave superconductor and an entirely different form of competing order, originating from the particle-hole channel[21]. However, up to now, the interplay between the SC gap and normal-state pseudogap in cuprate superconductors has not been treated starting from a microscopic SC theory, therefore no general consensus for the normal-state pseudogap has been reached yet on the its origin, its role in the onset of superconductivity itself, and not even on its evolution across the phase diagram of cuprate superconductors[2].

In our earlier work, a kinetic energy driven SC mechanism has been developed[22], where the interaction between charge carriers and spins directly from the kinetic energy by exchanging spin excitations induces a d-wave charge carrier pairing state, and then their condensation reveals the SC ground-state. In particular, this SC-state is controlled by both the SC gap and quasiparticle coherence, which leads to that the maximal SC transition temperature occurs around the optimal doping, and then decreases in both the underdoped and overdoped regimes. Within this kinetic energy driven SC mechanism, we have discussed the low energy electronic structure[23], quasiparticle transport[24], and Meissner effect[25], and qualitatively reproduced some main features of the corresponding experimental results of cuprate superconductors in the SC-state. In this paper, we study the interplay between the SC gap and normal-state pseudogap in cuprate superconductors based on this kinetic energy driven SC mechanism[22], where one of our main results is that the interaction between charge carriers and spins directly from the kinetic energy by exchanging spin excitations

induces the normal-state pseudogap state in the particle-hole channel and SC-state in the particle-particle channel, therefore there is a coexistence of the SC gap and normal-state pseudogap in the whole SC dome. Our results also show that both the normal-state pseudogap and SC gap are dominated by one energy scale, and the normal-state pseudogap also is a necessary ingredient for superconductivity.

In cuprate superconductors, the characteristic feature is the presence of the CuO_2 plane[4]. In this case, it is commonly accepted that the essential physics of the doped CuO_2 plane[26] is captured by the t-J model on a square lattice,

$$H = -t\sum_{i\hat{\eta}\sigma} C^{\dagger}_{i\sigma}C_{i+\hat{\eta}\sigma} + t'\sum_{i\hat{\tau}\sigma} C^{\dagger}_{i\sigma}C_{i+\hat{\tau}\sigma} + \mu\sum_{i\sigma} C^{\dagger}_{i\sigma}C_{i\sigma} + J\sum_{i\hat{\eta}} \mathbf{S}_i\cdot\mathbf{S}_{i+\hat{\eta}}, \quad (1)$$

where $\hat{\eta}=\pm\hat{x},\pm\hat{y}$, $\hat{\tau}=\pm\hat{x}\pm\hat{y}$, $C^{\dagger}_{i\sigma}$ ($C_{i\sigma}$) is the electron creation (annihilation) operator, $\mathbf{S}_i=(S^x_i,S^y_i,S^z_i)$ are spin operators, and μ is the chemical potential. This t-J model (1) is subject to an important local constraint $\sum_{\sigma}C^{\dagger}_{i\sigma}C_{i\sigma}\leq 1$ to avoid the double occupancy. To incorporate this electron single occupancy local constraint, the charge-spin separation (CSS) fermion-spin theory[27] has been proposed, where the physics of no double occupancy is taken into account by representing the electron as a composite object created by $C_{i\uparrow}=h^{\dagger}_{i\uparrow}S^{-}_i$ and $C_{i\downarrow}=h^{\dagger}_{i\downarrow}S^{+}_i$, with the spinful fermion operator $h_{i\sigma}=e^{-i\Phi_{i\sigma}}h_i$ that describes the charge degree of freedom of the electron together with some effects of spin configuration rearrangements due to the presence of the doped hole itself (charge carrier), while the spin operator S_i represents the spin degree of freedom of the electron, then the electron single occupancy local constraint is satisfied in analytical calculations. In this CSS fermion-spin representation, the t-J model (1) can be expressed as[22,27],

$$\begin{aligned} H &= t\sum_{i\hat{\eta}}(h^{\dagger}_{i+\hat{\eta}\uparrow}h_{i\uparrow}S^{+}_iS^{-}_{i+\hat{\eta}} + h^{\dagger}_{i+\hat{\eta}\downarrow}h_{i\downarrow}S^{-}_iS^{+}_{i+\hat{\eta}}) \\ &- t'\sum_{i\hat{\tau}}(h^{\dagger}_{i+\hat{\tau}\uparrow}h_{i\uparrow}S^{+}_iS^{-}_{i+\hat{\tau}} + h^{\dagger}_{i+\hat{\tau}\downarrow}h_{i\downarrow}S^{-}_iS^{+}_{i+\hat{\tau}}) \\ &- \mu\sum_{i\sigma}h^{\dagger}_{i\sigma}h_{i\sigma} + J_{\rm eff}\sum_{i\hat{\eta}}\mathbf{S}_i\cdot\mathbf{S}_{i+\hat{\eta}}, \end{aligned} \quad (2)$$

where $J_{\rm eff}=(1-\delta)^2J$, and $\delta=\langle h^{\dagger}_{i\sigma}h_{i\sigma}\rangle=\langle h^{\dagger}_ih_i\rangle$ is the doping concentration. As a consequence, the kinetic energy in the t-J model has been transferred as the interaction between charge carriers and spins, which reflects that even the kinetic energy in the t-J model has strong Coulombic contribution due to the restriction of single occupancy of a given site.

As in the conventional superconductors, the SC-state in cuprate superconductors is also characterized by the electron Cooper pairs, forming SC quasiparticles[28]. On the other hand, the angle resolved photoemission spectroscopy measurements[4,29] have shown that in the real space the gap function and pairing force have a range of one lattice spacing. In this case, the order parameter for the electron Cooper pair can be expressed as[22],

$$\begin{aligned} \Delta &= \langle C^{\dagger}_{i\uparrow}C^{\dagger}_{i+\hat{\eta}\downarrow} - C^{\dagger}_{i\downarrow}C^{\dagger}_{i+\hat{\eta}\uparrow}\rangle \\ &= \langle h_{i\uparrow}h_{i+\hat{\eta}\downarrow}S^{+}_iS^{-}_{i+\hat{\eta}} - h_{i\downarrow}h_{i+\hat{\eta}\uparrow}S^{-}_iS^{+}_{i+\hat{\eta}}\rangle. \end{aligned} \quad (3)$$

In the doped regime without AFLRO, the charge carriers move in the background of the disordered spin liquid state, where the spin correlation functions $\langle S^{+}_iS^{-}_{i+\hat{\eta}}\rangle=\langle S^{-}_iS^{+}_{i+\hat{\eta}}\rangle=\chi_1$, and then the SC gap parameter in Eq. (3) can be written as $\Delta=-\chi_1\Delta_{\rm h}$, with the charge carrier pair gap parameter,

$$\Delta_{\rm h}=\langle h_{i+\hat{\eta}\downarrow}h_{i\uparrow} - h_{i+\hat{\eta}\uparrow}h_{i\downarrow}\rangle, \quad (4)$$

which shows that the SC gap parameter is closely related to the charge carrier pair gap parameter, therefore the essential physics in the SC-state is dominated by the corresponding one in the charge carrier pairing state. However, in the extreme low doped regime with AFLRO, where the spin correlation functions $\langle S^{+}_iS^{-}_{i+\hat{\eta}}\rangle\neq\langle S^{-}_iS^{+}_{i+\hat{\eta}}\rangle$, and the conduct is disrupted by AFLRO. Therefore in the following discussions, we only focus on the case without AFLRO as our previous studies[22].

The interaction between charge carriers and spins in the t-J model (2) is quite strong, and we[22] have shown in terms of Eliashberg's strong coupling theory[30] that in the case without AFLRO, this interaction can induce the d-wave electron Cooper pairing state by exchanging spin excitations. Following our previous discussions[22], the self-consistent equations that satisfied by the full charge carrier diagonal and off-diagonal Green's functions are obtained as,

$$\begin{aligned} g(k) &= g^{(0)}(k) + g^{(0)}(k)[\Sigma^{(\rm h)}_1(k)g(k) \\ &- \Sigma^{(\rm h)}_2(-k)\Gamma^{\dagger}(k)], \end{aligned} \quad (5a)$$

$$\begin{aligned} \Gamma^{\dagger}(k) &= g^{(0)}(-k)[\Sigma^{(\rm h)}_1(-k)\Gamma^{\dagger}(-k) \\ &+ \Sigma^{(\rm h)}_2(-k)g(k)], \end{aligned} \quad (5b)$$

respectively, where the four-vector notation $k=(\mathbf{k},i\omega_n)$, and the charge carrier mean-field (MF) Green's function[22,23], $g^{(0)-1}(k)=i\omega_n-\xi_{\mathbf{k}}$, with the MF charge carrier spectrum $\xi_{\mathbf{k}}=Zt\chi_1\gamma_{\mathbf{k}}-Zt'\chi_2\gamma'_{\mathbf{k}}-\mu$, the spin correlation function $\chi_2=\langle S^{+}_iS^{-}_{i+\hat{\tau}}\rangle$, $\gamma_{\mathbf{k}}=(1/Z)\sum_{\hat{\eta}}e^{i\mathbf{k}\cdot\hat{\eta}}$, $\gamma'_{\mathbf{k}}=(1/Z)\sum_{\hat{\tau}}e^{i\mathbf{k}\cdot\hat{\tau}}$, and Z is the number of the nearest neighbor or next-nearest neighbor sites, while the self-energies $\Sigma^{(\rm h)}_1(k)$ in the particle-hole channel and $\Sigma^{(\rm h)}_2(k)$ in the particle-particle channel have been evaluated from

the spin bubble as[22],

$$\Sigma_1^{(h)}(k) = \frac{1}{N^2}\sum_{\mathbf{p},\mathbf{p}'}\Lambda^2_{\mathbf{p}+\mathbf{p}'+\mathbf{k}} \times \frac{1}{\beta}\sum_{ip_m} g(p+k)\Pi(\mathbf{p},\mathbf{p}',ip_m), \tag{6a}$$

$$\Sigma_2^{(\mathrm{h})}(k) = \frac{1}{N^2}\sum_{\mathbf{p},\mathbf{p}'}\Lambda^2_{\mathbf{p}+\mathbf{p}'+\mathbf{k}} \times \frac{1}{\beta}\sum_{ip_m} \Gamma^{\dagger}(-p-k)\Pi(\mathbf{p},\mathbf{p}',ip_m), \tag{6b}$$

with $\Lambda_{\mathbf{k}} = Zt\gamma_{\mathbf{k}} - Zt'\gamma'_{\mathbf{k}}$, and the spin bubble,

$$\Pi(\mathbf{p},\mathbf{p}',ip_m) = \frac{1}{\beta}\sum_{ip'_m} D^{(0)}(p')D^{(0)}(p'+p), \tag{7}$$

where $p = (\mathbf{p}, ip_m)$, $p' = (\mathbf{p}', ip'_m)$, and the MF spin Green's function, $D^{(0)-1}(p) = [(ip_m)^2 - \omega^2_{\mathbf{p}}]/B_{\mathbf{p}}$, with the MF spin excitation spectrum $\omega_{\mathbf{p}}$ and $B_{\mathbf{p}}$ have been given in Ref. 23.

Since the pairing force and charge carrier pair gap have been incorporated into the self-energy $\Sigma_2^{(\mathrm{h})}(k)$, it is called as the effective charge carrier pair gap $\bar{\Delta}_{\mathrm{h}}(k) = \Sigma_2^{(\mathrm{h})}(k)$. On the other hand, the self-energy $\Sigma_1^{(\mathrm{h})}(k)$ renormalizes the MF charge carrier spectrum[31]. In particular, $\Sigma_2^{(\mathrm{h})}(k)$ is an even function of $i\omega_n$, while $\Sigma_1^{(\mathrm{h})}(k)$ is not. In our previous discussions[22], $\Sigma_1^{(\mathrm{h})}(k)$ has been broken up into its symmetric and antisymmetric parts as, $\Sigma_1^{(\mathrm{h})}(k) = \Sigma_{1\mathrm{e}}^{(\mathrm{h})}(k) + i\omega_n\Sigma_{1\mathrm{o}}^{(\mathrm{h})}(k)$, then both $\Sigma_{1\mathrm{e}}^{(\mathrm{h})}(k)$ and $\Sigma_{1\mathrm{o}}^{(\mathrm{h})}(k)$ are an even function of $i\omega_n$. In this case, we[22] have defined the charge carrier quasiparticle coherent weight $Z^{-1}_{\mathrm{hF}}(k) = 1 - \mathrm{Re}\Sigma_{1\mathrm{o}}^{(\mathrm{h})}(k)$. In the static limit approximation for the effective charge carrier pair gap and quasiparticle coherent weight, i.e., $\bar{\Delta}_{\mathrm{h}}(\mathbf{k}) = \bar{\Delta}_{\mathrm{h}}\gamma^{(\mathrm{d})}_{\mathbf{k}}$ with $\gamma^{(\mathrm{d})}_{\mathbf{k}} = (\cos k_x - \cos k_y)/2$, and $Z^{-1}_{\mathrm{hF}} = 1 - \mathrm{Re}\Sigma_{1\mathrm{o}}^{(\mathrm{h})}(\mathbf{k},\omega = 0)|_{\mathbf{k}=[\pi,0]}$, the BCS-like charge carrier diagonal and off-diagonal Green's functions with the d-wave symmetry have been obtained[22,23], although the pairing mechanism is driven by the kinetic energy by exchanging spin excitations. With the help of these charge carrier diagonal and off-diagonal Green's functions, $\Sigma_1^{(\mathrm{h})}(k)$ and $\bar{\Delta}_{\mathrm{h}}(\mathbf{k})$ in Eq. (6) have been obtained explicitly as[22,23],

$$\Sigma_1^{(h)}(k) = \frac{1}{N^2}\sum_{\mathbf{p}\mathbf{p}'\mu\nu\tau}\Lambda^2_{\mathbf{p}+\mathbf{p}'+\mathbf{k}}\frac{B_{\mathbf{p}'}B_{\mathbf{p}'+\mathbf{p}}}{8\omega_{\mu\mathbf{p}'}\omega_{\nu\mathbf{p}'+\mathbf{p}}} \times \frac{A_{\tau}(\mathbf{p}+\mathbf{k})F_{\mu\nu\tau}(\mathbf{p},\mathbf{p}',\mathbf{k})}{i\omega_n + \omega_{\nu\mathbf{p}'+\mathbf{p}} - \omega_{\mu\mathbf{p}'} - E^{(\tau)}_{\mathrm{h}\mathbf{p}+\mathbf{k}}}, \tag{8a}$$

$$\bar{\Delta}_{\mathrm{h}}(\mathbf{k}) = \frac{1}{N^2}\sum_{\mathbf{p}\mathbf{p}'\mu\nu\tau}\Lambda^2_{\mathbf{p}+\mathbf{p}'+\mathbf{k}}\frac{B_{\mathbf{p}'}B_{\mathbf{p}'+\mathbf{p}}}{8\omega_{\mu\mathbf{p}'}\omega_{\nu\mathbf{p}'+\mathbf{p}}} \times \frac{\bar{\Delta}_{\mathrm{hZ}}(\mathbf{p}+\mathbf{k})}{E^{(\tau)}_{\mathrm{h}\mathbf{p}+\mathbf{k}}}\frac{F_{\mu\nu\tau}(\mathbf{p},\mathbf{p}',\mathbf{k})}{\omega_{\nu\mathbf{p}'+\mathbf{p}} - \omega_{\mu\mathbf{p}'} - E^{(\tau)}_{\mathrm{h}\mathbf{p}+\mathbf{k}}}, \tag{8b}$$

where $F_{\mu\nu\tau}(\mathbf{p},\mathbf{p}',\mathbf{k}) = Z_{\mathrm{hF}}\{n_{\mathrm{F}}(E^{(\tau)}_{\mathrm{h}\mathbf{p}+\mathbf{k}})[n_{\mathrm{B}}(\omega_{\mu\mathbf{p}'}) - n_{\mathrm{B}}(\omega_{\nu\mathbf{p}'+\mathbf{p}})] + n_{\mathrm{B}}(\omega_{\nu\mathbf{p}'+\mathbf{p}})[1 + n_{\mathrm{B}}(\omega_{\mu\mathbf{p}'})]\}$, $\mu,\nu,\tau = 1,2$, $A_{\tau}(\mathbf{p}+\mathbf{k}) = 1 + \bar{\xi}_{\mathbf{p}+\mathbf{k}}/E^{(\tau)}_{\mathrm{h}\mathbf{p}+\mathbf{k}}$, $\omega_{1\mathbf{p}} = \omega_{\mathbf{p}}$, $\omega_{2\mathbf{p}} = -\omega_{\mathbf{p}}$, $E^{(1)}_{\mathrm{h}\mathbf{k}} = E_{\mathrm{h}\mathbf{k}}$, $E^{(2)}_{\mathrm{h}\mathbf{k}} = -E_{\mathrm{h}\mathbf{k}}$, the renormalized charge carrier excitation spectrum $\bar{\xi}_{\mathbf{k}} = Z_{\mathrm{hF}}\xi_{\mathbf{k}}$, the renormalized charge carrier pair gap $\bar{\Delta}_{\mathrm{hZ}}(\mathbf{k}) = Z_{\mathrm{hF}}\bar{\Delta}_{\mathrm{h}}(\mathbf{k})$, the charge carrier quasiparticle spectrum $E_{\mathrm{h}\mathbf{k}} = \sqrt{\bar{\xi}^2_{\mathbf{k}} + |\bar{\Delta}_{\mathrm{hZ}}(\mathbf{k})|^2}$, and $n_{\mathrm{B}}(\omega_{\mathbf{p}})$ and $n_{\mathrm{F}}(E_{h\mathbf{k}})$ are the boson and fermion distribution functions, respectively. In this case, the effective charge carrier pair gap parameter can be obtained in terms of Eq. (8) as[22,23],

$$\bar{\Delta}_{\mathrm{h}} = \frac{4}{N}\sum_{\mathbf{k}}\gamma^{(\mathrm{d})}_{\mathbf{k}}\bar{\Delta}_{\mathrm{h}}(\mathbf{k}). \tag{9}$$

However, the self-energy $\Sigma_1^{(\mathrm{h})}(k)$ in Eq. (8) also can be rewritten as,

$$\Sigma_1^{(h)}(k) = \frac{[2\bar{\Delta}_{\mathrm{pg}}(\mathbf{k})]^2}{i\omega_n + M_{\mathbf{k}}}, \tag{10}$$

where $M_{\mathbf{k}}$ is the energy spectrum of $\Sigma_1^{(\mathrm{h})}(k)$. As in the case of the effective charge carrier pair gap, since the interaction force and normal-state pseudogap have been incorporated into $\bar{\Delta}_{\mathrm{pg}}(\mathbf{k})$, it is called as the effective normal-state pseudogap. In this case, it is easy to find that in our previous static limit approximation[22,23] for $\Sigma_{1\mathrm{o}}^{(\mathrm{h})}(k)$, the quasiparticle coherent weight $Z^{-1}_{\mathrm{hF}} = \{1 + [2\bar{\Delta}_{\mathrm{pg}}(\mathbf{k})]^2/M^2_{\mathbf{k}}\}|_{\mathbf{k}=[\pi,0]}$, which reflects that the partial effect of the normal-state pseudogap has been contained in the quasiparticle coherent weight. Since the SC-state in the kinetic energy driven SC mechanism is controlled by both the SC gap and quasiparticle coherence[22], then in this sense, the normal-state pseudogap is a necessary ingredient for superconductivity. In the following discussions, we focus on the connection between the normal-state pseudogap and SC gap beyond our previous static limit approximation[22] for the self-energy $\Sigma_1^{(\mathrm{h})}(k)$, and show explicitly the two-gap feature in cuprate superconductors. Substituting the self-energy $\Sigma_1^{(\mathrm{h})}(k)$ in Eq. (10) into Eq. (5), we obtain the full charge carrier diagonal and off-diagonal Green's functions as,

$$\begin{aligned} g(k) &= \frac{1}{i\omega_n - \xi_{\mathbf{k}} - \Sigma_1^{(\mathrm{h})}(k) - \bar{\Delta}^2_{\mathrm{h}}(\mathbf{k})/[i\omega_n + \xi_{\mathbf{k}} + \Sigma_1^{(\mathrm{h})}(-k)]} \\ &= \frac{U^2_{1\mathrm{h}\mathbf{k}}}{i\omega_n - E_{1\mathrm{h}\mathbf{k}}} + \frac{V^2_{1\mathrm{h}\mathbf{k}}}{i\omega_n + E_{1\mathrm{h}\mathbf{k}}} + \frac{U^2_{2\mathrm{h}\mathbf{k}}}{i\omega_n - E_{2\mathrm{h}\mathbf{k}}} + \frac{V^2_{2\mathrm{h}\mathbf{k}}}{i\omega_n + E_{2\mathrm{h}\mathbf{k}}}, \end{aligned} \tag{11a}$$

$$\begin{aligned} \Gamma^{\dagger}(k) &= \frac{-\bar{\Delta}_{\mathrm{h}}(\mathbf{k})}{[i\omega_n - \xi_{\mathbf{k}} - \Sigma_1^{(\mathrm{h})}(k)][i\omega_n + \xi_{\mathbf{k}} + \Sigma_1^{(\mathrm{h})}(-k)] - \bar{\Delta}^2_{\mathrm{h}}(\mathbf{k})} \\ &= -\frac{\alpha_{1\mathbf{k}}\bar{\Delta}_{\mathrm{h}}(\mathbf{k})}{2E_{1\mathrm{h}\mathbf{k}}}\left(\frac{1}{i\omega_n - E_{1\mathrm{h}\mathbf{k}}} - \frac{1}{i\omega_n + E_{1\mathrm{h}\mathbf{k}}}\right) + \frac{\alpha_{2\mathbf{k}}\bar{\Delta}_{\mathrm{h}}(\mathbf{k})}{2E_{2\mathrm{h}\mathbf{k}}}\left(\frac{1}{i\omega_n - E_{2\mathrm{h}\mathbf{k}}} - \frac{1}{i\omega_n + E_{2\mathrm{h}\mathbf{k}}}\right), \end{aligned} \tag{11b}$$

where $\alpha_{1\mathbf{k}}=(E^{2}_{1h\mathbf{k}}-M^{2}_{\mathbf{k}})/(E^{2}_{1h\mathbf{k}}-E^{2}_{2h\mathbf{k}})$, $\alpha_{2\mathbf{k}}=(E^{2}_{2h\mathbf{k}}-M^{2}_{\mathbf{k}})/(E^{2}_{1h\mathbf{k}}-E^{2}_{2h\mathbf{k}})$, and there are four branches of the charge carrier quasiparticle spectrum due to the presence of the normal-state pseudogap and SC gap, $E_{1h\mathbf{k}}$, $-E_{1h\mathbf{k}}$, $E_{2h\mathbf{k}}$, and $-E_{2h\mathbf{k}}$, with $E_{1h\mathbf{k}}=\sqrt{[\Omega_{\mathbf{k}}+\Theta_{\mathbf{k}}]/2}$, $E_{2h\mathbf{k}}=\sqrt{[\Omega_{\mathbf{k}}-\Theta_{\mathbf{k}}]/2}$, and the kernel functions,

$$\Omega_{\mathbf{k}} = \xi^{2}_{\mathbf{k}}+M^{2}_{\mathbf{k}}+8\bar{\Delta}^{2}_{\rm pg}(\mathbf{k})+\bar{\Delta}^{2}_{\rm h}(\mathbf{k}), \tag{12a}$$

$$\Theta_{\mathbf{k}} = \sqrt{(\xi^{2}_{\mathbf{k}}-M^{2}_{\mathbf{k}})\beta_{1\mathbf{k}}+16\bar{\Delta}^{2}_{\rm pg}(\mathbf{k})\beta_{2\mathbf{k}}+\bar{\Delta}^{4}_{\rm h}(\mathbf{k})}, \tag{12b}$$

with $\beta_{1\mathbf{k}}=\xi^{2}_{\mathbf{k}}-M^{2}_{\mathbf{k}}+2\bar{\Delta}^{2}_{\rm h}(\mathbf{k})$, $\beta_{2\mathbf{k}}=(\xi_{\mathbf{k}}-M_{\mathbf{k}})^{2}+\bar{\Delta}^{2}_{\rm h}(\mathbf{k})$, while the coherence factors,

$$U^{2}_{1h\mathbf{k}} = \frac{1}{2}\left[\alpha_{1\mathbf{k}}\left(1+\frac{\xi_{\mathbf{k}}}{E_{1h\mathbf{k}}}\right)-\alpha_{3\mathbf{k}}\left(1+\frac{M_{\mathbf{k}}}{E_{1h\mathbf{k}}}\right)\right], \tag{13a}$$

$$V^{2}_{1h\mathbf{k}} = \frac{1}{2}\left[\alpha_{1\mathbf{k}}\left(1-\frac{\xi_{\mathbf{k}}}{E_{1h\mathbf{k}}}\right)-\alpha_{3\mathbf{k}}\left(1-\frac{M_{\mathbf{k}}}{E_{1h\mathbf{k}}}\right)\right], \tag{13b}$$

$$U^{2}_{2h\mathbf{k}} = -\frac{1}{2}\left[\alpha_{2\mathbf{k}}\left(1+\frac{\xi_{\mathbf{k}}}{E_{2h\mathbf{k}}}\right)-\alpha_{3\mathbf{k}}\left(1+\frac{M_{\mathbf{k}}}{E_{2h\mathbf{k}}}\right)\right], \tag{13c}$$

$$V^{2}_{2h\mathbf{k}} = -\frac{1}{2}\left[\alpha_{2\mathbf{k}}\left(1-\frac{\xi_{\mathbf{k}}}{E_{2h\mathbf{k}}}\right)-\alpha_{3\mathbf{k}}\left(1-\frac{M_{\mathbf{k}}}{E_{2h\mathbf{k}}}\right)\right], \tag{13d}$$

satisfy the sum rule: $U^{2}_{1h\mathbf{k}}+V^{2}_{1h\mathbf{k}}+U^{2}_{2h\mathbf{k}}+V^{2}_{2h\mathbf{k}}=1$, where $\alpha_{3\mathbf{k}}=[2\bar{\Delta}_{\rm pg}(\mathbf{k})]^{2}/(E^{2}_{1h\mathbf{k}}-E^{2}_{2h\mathbf{k}})$, and the corresponding effective normal-state pseudogap $\bar{\Delta}_{\rm pg}(\mathbf{k})$ and energy spectrum $M_{\mathbf{k}}$ in Eq. (10) can be obtained explicitly in terms of the self-energy $\Sigma^{(h)}_{1}(k)$ in Eq. (8) as,

$$\bar{\Delta}_{\rm pg}(\mathbf{k}) = \frac{L_{2}(\mathbf{k})}{2\sqrt{L_{1}(\mathbf{k})}}, \tag{14a}$$

$$M_{\mathbf{k}} = \frac{L_{2}(\mathbf{k})}{L_{1}(\mathbf{k})}, \tag{14b}$$

with $L_{1}(\mathbf{k})$ and $L_{2}(\mathbf{k})$ are given by,

$$L_{1}(\mathbf{k}) = \frac{1}{N^{2}}\sum_{\mathbf{pp'}\mu\nu\tau}\Lambda^{2}_{\mathbf{p}+\mathbf{p'}+\mathbf{k}}\frac{B_{\mathbf{p'}}B_{\mathbf{p'}+\mathbf{p}}}{8\omega_{\mu\mathbf{p'}}\omega_{\nu\mathbf{p'}+\mathbf{p}}} \times \frac{A_{\tau}(\mathbf{p}+\mathbf{k})F_{\mu\nu\tau}(\mathbf{p},\mathbf{p'},\mathbf{k})}{[\omega_{\nu\mathbf{p'}+\mathbf{p}}-\omega_{\mu\mathbf{p'}}-E^{(\tau)}_{h\mathbf{p}+\mathbf{k}}]^{2}}, \tag{15a}$$

$$L_{2}(\mathbf{k}) = \frac{1}{N^{2}}\sum_{\mathbf{pp'}\mu\nu\tau}\Lambda^{2}_{\mathbf{p}+\mathbf{p'}+\mathbf{k}}\frac{B_{\mathbf{p'}}B_{\mathbf{p'}+\mathbf{p}}}{8\omega_{\mu\mathbf{p'}}\omega_{\nu\mathbf{p'}+\mathbf{p}}} \times \frac{A_{\tau}(\mathbf{p}+\mathbf{k})F_{\mu\nu\tau}(\mathbf{p},\mathbf{p'},\mathbf{k})}{\omega_{\nu\mathbf{p'}+\mathbf{p}}-\omega_{\mu\mathbf{p'}}-E^{(\tau)}_{h\mathbf{p}+\mathbf{k}}}. \tag{15b}$$

In this case, it then is straightforward to obtain the effective normal-state pseudogap parameter from Eq. (14) as $\bar{\Delta}_{\rm pg}=(1/N)\sum_{\mathbf{k}}\bar{\Delta}_{\rm pg}(\mathbf{k})$.

Now we are ready to discuss the interplay between the SC gap and normal-state pseudogap. In cuprate superconductors, although the values of J, t, and t' are believed to vary somewhat from compound to compound[4], however, as a qualitative discussion, the commonly used parameters in this paper are chosen as $t/J=2.5$, $t'/t=0.3$, and $J=110$meV. In this case, the effective normal-state pseudogap parameter ($2\bar{\Delta}_{\rm pg}$) (solid line) and the effective charge carrier pair gap parameter ($2\bar{\Delta}_{\rm h}$) (dashed line) as a function of doping for temperature $T=0.002J$ are plotted in Fig. 1 in comparison with the corresponding experimental data[2] of cuprate superconductors (inset). Obviously, the two-gap feature observed on cuprate superconductors[2] is qualitatively reproduced. In particular, $\bar{\Delta}_{\rm h}$ increases with increasing the doping concentration in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime[22,23]. However, in contrast to the case of $\bar{\Delta}_{\rm h}$ in the underdoped regime, $\bar{\Delta}_{\rm pg}$ smoothly increases with decreasing the doping concentration in the underdoped regime, this leads to that $\bar{\Delta}_{\rm pg}$ is much larger than $\bar{\Delta}_{\rm h}$ in the underdoped regime. Moreover, $\bar{\Delta}_{\rm pg}$ seems to merge with $\bar{\Delta}_{\rm h}$ in the overdoped regime, eventually disappearing together with superconductivity at the doping concentrations larger than $\delta\sim 0.27$.

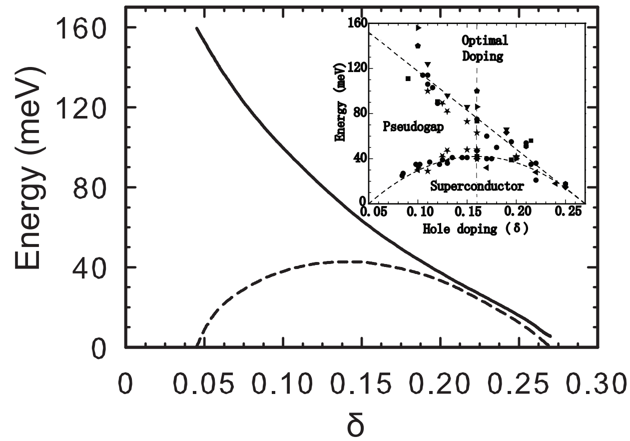

FIG. 1: The effective normal-state pseudogap parameter ($2\bar{\Delta}_{\rm R}$) (solid line) and effective charge carrier pair gap parameter ($2\bar{\Delta}_{\rm h}$) (dashed line) as a function of doping for temperature $T=0.002J$ with parameters $t/J=2.5$, $t'/t=0.3$, and $J=110$meV. Inset: the corresponding experimental data of cuprate superconductors taken from Ref. 2.

As in the temperature dependence of the SC gap, this normal-state pseudogap is also temperature dependent. In particular, in the given doping concentration, the normal-state pseudogap vanishes when temperature reaches the normal-state pseudogap crossover temperature T^{*}. To show this doping dependence of T^{*} clearly, we have made a series of calculations for T^{*} at different doping concentrations, and the results of T^{*} (solid line) and SC transition temperature $T_{\rm c}$ (dashed line) as a function of doping are plotted in Fig. 2 in comparison with the experimental data of cuprate superconductors[2]. In corresponding to the results of the doping dependence of $\bar{\Delta}_{\rm h}$ and $\bar{\Delta}_{\rm h}$ as shown in Fig. 1, T^{*} is much larger than T_{c} in the underdoped regime, then it smoothly decreases with increasing the doping concentration. Moreover, both T^{*} and T_{c} converge to the end of the SC dome,

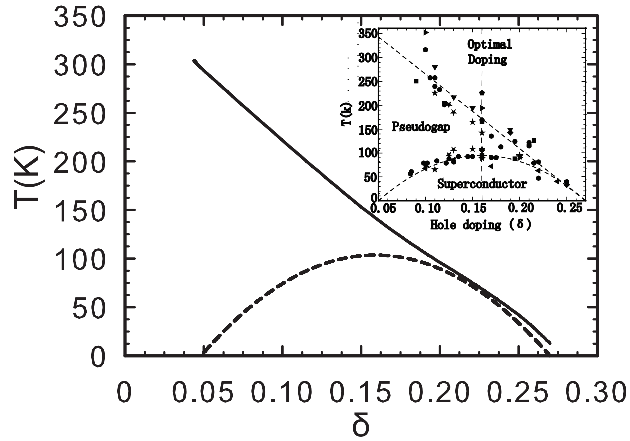

FIG. 2: The normal-state pseudogap crossover temperature T^* (solid line) and superconducting transition temperature T_c (dashed line) as a function of doping for parameters $t/J = 2.5$, $t'/t = 0.3$, and $J = 110$meV. Inset: the corresponding experimental data of cuprate superconductors taken from Ref. 2.

in qualitative agreement with the experimental results[2].

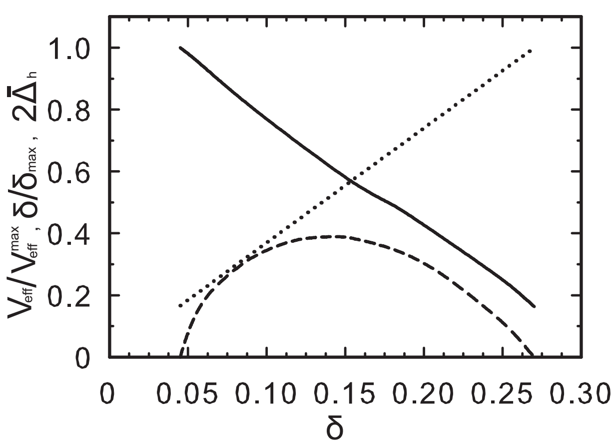

FIG. 3: The strength of the attractive interaction (solid line), the doping (dotted line), and the effective charge carrier pair gap parameter ($2\bar{\Delta}_{\rm h}$) (dashed line) as a function of doping for temperature $T = 0.002J$ with parameters $t/J = 2.5$, $t'/t = 0.3$, and $J = 110$meV.

Our present results in Fig. 1 and Fig. 2 show clearly that there are two coexisting energy gaps in the whole SC dome: one associated with a direct measure of the binding energy of the two electrons forming a Cooper pair[22], while the other with the anomalous normal-state properties[14,20]. Within the kinetic energy driven SC mechanism[22], the essential physics of this two-gap feature in cuprate superconductors can be attributed to the doping and temperature dependence of the charge carrier interactions in the particle-hole and particle-particle channels directly from the kinetic energy by exchanging spin excitations. The parent compounds of cuprate superconductors are the Mott insulators[4] as we have mentioned above, when charge carriers are doped into a Mott insulator, there is a gain in the kinetic energy per charge carrier proportional to t due to hopping, however, at the same time, the magnetic energy is decreased, costing an energy of approximately J per site, therefore the doped charge carriers into a Mott insulator can be considered as a competition between the kinetic energy (δt) and magnetic energy (J). This leads to the spin excitation spectral strength decreases with increasing the doping concentration. In the particle-particle channel, the interaction between the charge carriers mediated by spin excitations is attractive, then the system of charge carriers forms pairs of bound charge carriers[22]. Since the pairing force and charge carrier pair gap have been incorporated into the *effective* charge carrier pair gap[22] as mentioned above, the strength $V_{\rm eff}$ of this attractive interaction can be obtained in terms of the *effective* charge carrier pair gap parameter $\bar{\Delta}_{\rm h}$ and charge carrier pair gap parameter $\Delta_{\rm h}$ as $V_{\rm eff} = \bar{\Delta}_{\rm h}/\Delta_{\rm h}$, where $\Delta_{\rm h}$ is evaluated in terms of the charge carrier off-diagonal Green's function, and has been given in Refs. 22 and 23. In this case, a decrease of the spin excitation spectral strength with increasing the doping concentration leads to a decrease of the coupling strength $V_{\rm eff}$ with increasing the doping concentration. To see this point clearly, we have calculated the doping dependence of $V_{\rm eff}$, and the results of $V_{\rm eff}/V_{\rm eff}^{\rm max}$ (solid line), $\delta/\delta_{\rm max}$ (dotted line), and $2\bar{\Delta}_{\rm h}$ (dashed line) as a function of doping for $T = 0.002J$ are plotted in Fig. 3, where $V_{\rm eff}^{\rm max} = V_{\rm eff}|_{\delta=0.045}$ is the value of $V_{\rm eff}$ at the starting point of the SC dome, while $\delta_{\rm max} = 0.27$ is the doping concentration at the end point of the SC dome. Our results in Fig. 3 show clearly that the coupling strength $V_{\rm eff}$ smoothly decreases upon increasing the doping concentration from a strong-coupling case in the underdoped regime to a weak-coupling side in the overdoped regime. Moreover, in the underdoped regime, the coupling strength $V_{\rm eff}$ is very strong to form the charge carrier pairs for the most charge carriers, and therefore the number of the charge carrier pairs increases with increasing the doping concentration, which leads to that the charge carrier pair gap parameter and SC transition temperature increase with increasing the doping concentration. However, in the overdoped regime, the coupling strength $V_{\rm eff}$ is relatively weak. In this case, not all charge carriers can be bound to form the charge carrier pairs by this weakly attractive interaction, and therefore the number of the charge carrier pairs decreases with increasing the doping concentration, this leads to that the charge carrier pair gap parameter and SC transition temperature decrease with increasing the doping concentration. In particular, the optimal doping is a balance point, where the number of the charge carrier pairs and coupling strength $V_{\rm eff}$ are optimally matched. This is why the maximal charge carrier pair gap parameter and SC transition temperature occur around the optimal doping, and then decreases in both the underdoped and overdoped regimes[22]. On the other hand, in the particle-hole channel, the charge carriers also interact by exchang-

ing spin excitations as in the case in the particle-particle channel. This interaction in the particle-hole channel reduces the charge carrier quasiparticle bandwidth[31], and therefore the energy scale of the electron quasiparticle band is controlled by the magnetic interaction J. In this case, the effective normal-state pseudogap parameter $\bar{\Delta}_{\rm pg}$ and normal-state pseudogap crossover temperature T^* originated from the self-energy $\Sigma^{(h)}_1(k)$ have the same doping dependent behavior of $V_{\rm eff}$, i.e., they reaches the maximum at the starting point of the SC dome, and then decreases with increasing the doping concentration, eventually disappearing at the end point of the SC dome. Furthermore, since the charge carrier interactions in both the particle-hole and particle-particle channels are mediated by the same spin excitations as shown in Eq. (6), therefore all these charge carrier interactions are controlled by the same magnetic interaction J. In this sense, both the normal-state pseudogap and SC gap in the phase diagram of cuprate superconductors as shown in Fig. 1 are dominated by one energy scale. Moreover, our present theory starting from the t-J model also shows that both the normal-state pseudogap and SC gap in cuprate superconductors are the result of the strong electron correlation.

In conclusion, we have discussed the interplay between the SC gap and normal-state pseudogap in cuprate superconductors based on the kinetic energy driven SC mechanism. Our results show that the interaction between charge carriers and spins directly from the kinetic energy by exchanging spin excitations induces the normal-state pseudogap state in the particle-hole channel and SC-state in the particle-particle channel, therefore there is a coexistence of the SC gap and normal-state pseudogap in the whole SC dome. This normal-state pseudogap is closely related to the quasiparticle coherent weight, and is a necessary ingredient for superconductivity in cuprate superconductors. Furthermore, our results also show that both the normal-state pseudogap and SC gap are dominated by one energy scale, and they are the result of the strong electron correlation.

Although the normal-state pseudogap phenomenon and its relevance to superconductivity can also be discussed starting directly from some phenomenological theories[15–21], however, we in this paper are primarily interested in exploring the general notion of the interplay between the SC gap and normal-state pseudogap in the kinetic energy driven cuprate superconductors. Experimentally, the SC transition is a true transition with all necessary peculiarities in thermodynamic quantities, whereas the normal-state pseudogap transition is just a crossover, and its effect is reflected in the anomalous normal-state properties. Recently, the specific-heat measurement[32] on cuprate superconductors shows that the specific-heat has a humplike anomaly near the SC transition temperature T_c, and behaves as a long tail which persists far into the normal-state in the underdoped regime, but in the heavily overdoped regime the anomaly ends sharply just near T_c. In this case, we[33] have studied the doping dependence of the thermodynamic properties in cuprate superconductors within the present framework, and the results show that this humplike anomaly of the specific-heat near SC transition temperature in the underdoped regime can be attributed to the emergence of the normal-state pseudogap[33]. Furthermore, we[34] have also discussed the evolution of the Fermi arcs with doping and temperature within the present framework. In particular, we show that when the temperature $T \sim 0$, the Fermi arc in the underdoped regime is converged around the nodal point, however, with increasing temperatures, it collapses almost linearly with temperature T, in qualitative agreement with the experimental results[35]. These and the related results will be presented elsewhere.

Finally, we have noted that as for the normal-state pseudogap, which grows upon underdoping, it seems natural to seek a connection to the physics of the AF insulating parent compound[2]. However, at the half-filling, the t-J model is reduced as the AF Heisenberg model with an AFLRO. In particular, this AFLRO is kept until the the extreme low doped regime ($\delta < 0.045$)[36]. As we have mentioned in Eq. (3), the conduct is disrupted by AFLRO in this extreme low doped regime, and then our present theory based on the disordered spin liquid state is invalid[22]. In this case, an important issue is how to extend the present theory in the norma-state for the doped regime without AFLRO to the case in the extreme low doped regime with AFLRO for a proper description of the connection between the finite doping normal-state pseudogap and the zero-doping quasiparticle dispersion. These and the related issues are under investigation now.

Acknowledgments

The authors would like to thank Dr. Yu Lan for the help in the numerical calculations. This work was supported by the National Natural Science Foundation of China under Grant No. 11074023, and the funds from the Ministry of Science and Technology of China under Grant Nos. 2011CB921700 and 2012CB821403.

* To whom correspondence should be addressed, E-mail: spfeng@bnu.edu.cn

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